Unitary Symmetry and the $K \to 2\pi$ Decay*

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It is shown that all the $K \to 2\pi$ decays are forbidden in the limit of complete SU_3 symmetry of strong interactions, provided only (a) that both K and π mesons belong to the same octet and (b) that the nonleptonic decay interactions belong to the CP-invariant representation (the irreducible representation whose $Y=Q=0$ members are CP-invariant). This is more general than the conclusion of Cabibbo and Gell-Mann. In their proof, the nonleptonic decay Lagrangian is assumed to belong to a CP-invariant octet. This result suggests that the observed large decay rate of $K_1^0 \to 2\pi$ decay is due to the symmetry-breaking interactions and its large Q value. Finally, a brief comment is given concerning the $(K_1^0 \to 2\pi)-(K^+ \to 2\pi)$ puzzle in the $\Delta I=\frac{1}{2}$ rule. It is shown that, if one assumes the $\Delta I=\frac{1}{2}$ rule in addition to the above hypothesis (b), the $K^\pm \to 2\pi$ decay can occur only through the electromagnetic corrections and its effective decay Lagrangian should belong to the representations 10 and $\overline{10}$.

It has been shown by Cabibbo[1] and Gell-Mann[2] that all the $K \to 2\pi$ decays are forbidden if the Lagrangian for the nonleptonic decay process behaves as a member of an octet under the transformations of the group SU_3. The present note carries the argument further and shows that this result is not peculiar to the octet hypothesis of the decay Lagrangian; rather it has a more general validity in the SU_3 scheme. Finally, a brief remark will be added concerning the $\Delta I=\frac{1}{2}$ rule and the $(K_1^0 \to 2\pi)-(K^+ \to 2\pi)$ puzzle. The basic hypotheses in the following arguments are (a) that *both K and π mesons are members of the same pseudoscalar meson octet*,[3,4] and (b) that *the nonleptonic decay interactions belong to the CP-invariant representations*. The precise content of hypothesis (b) will be clarified later. In addition to the above, the strong interactions are assumed to be completely SU_3 symmetric. All of these hypotheses are assumed also in Refs. 1 and 2, but the essential point is that the present note does not assume the octet behavior of nonleptonic decay Lagrangian.

Consider the transition between a spurion and the $K\pi\pi$ system, which is responsible for both the $K \to 2\pi$ and the $\pi \to K\pi$ "decays." The relevant state of the $K\pi\pi$ system is totally symmetric among the three particles. This and the hypothesis (a) imply that the spurion, and thus the Lagrangian responsible for the $K \to 2\pi$ decay, should belong to any one of the representations 8, 10, $\overline{10}$, 27, and 64, or any linear combination of them. (The singlet is excluded because of the $\Delta S=\pm 1$ character of the process.)

Now the nonleptonic decays are characterized by $\Delta S=\pm 1$ and $\Delta Q=0$, or equivalently by $\Delta U_3=\pm 1$ with $\Delta Q=0$, where S, Q, and U_3 are, respectively, the strangeness, charge, and the third component of the U spin.[5] [In terms of the unitary spin defined by Gell-Mann,[3] $U_1=F_6$, $U_2=F_7$, and $U_3=\frac{1}{2}(\sqrt{3}F_8-F_3)$.] Therefore, the decay Lagrangian can be expressed as a linear combination of the terms $L_u^{(+)}$ and $L_u^{(-)}$, whose transformation property under the rotation in the U-spin space are

$$L_u^{(\pm)} \propto \Psi(u,1) \pm \Psi(u,-1). \qquad (1)$$

Here, $\Psi(u,u_3)$ is the normalized eigenvector operator of the U spin, u_3 and u being, respectively, the eigenvalues of U_3 and of the magnitude of U.[6] The pair $\Psi(u,1)$ and $\Psi(u,-1)$, and thus the pair $L_u^{(+)}$ and $L_u^{(-)}$, belong to the same U-spin multiplet (with $Q=0$) in the same SU_3 multiplet. There may be several kinds of such pairs $(L_u^{(+)},L_u^{(-)})$ which belong to different representations and/or different u values.

Each pair $(L_u^{(+)},L_u^{(-)})$ has its partner $L_u^{(0)}$ which behaves as $\Psi(u,0)$ under the rotation in the U-spin space and, of course, belongs to the same representation. *It is assumed here that this $L_u^{(0)}$ is CP invariant.* This is the precise content of the hypothesis (b) of the present note. The word "CP-invariant representation" in the hypothesis (b) means the *irreducible* representation whose $Q=0$, $U_3=0$ members are CP invariant. This requirement is more restrictive than the assumption of simple CP invariance of whole Lagrangian, but it is actually satisfied by, for example, the general CP-invariant symmetric current-current form of nonleptonic decay interaction. By the above definition,

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[6] Here the phase convention adopted for the eigenvectors $\Psi(u,u_3)$ is

$$(U_1 \pm iU_2)\Psi(u,u_3) = [(u \mp u_3)(u \pm u_3+1)]^{1/2}\Psi(u,u_3 \pm 1).$$

As for the members of the pseudoscalar meson octet, the assignment is

$$(u_3=1,0,-1) = (-K^0, -\tfrac{1}{2}(\pi^0-\sqrt{3}\eta), \bar{K}^0) \text{ for the } U\text{-spin triplet,}$$

$$(u_3=\tfrac{1}{2},-\tfrac{1}{2}) = \begin{Bmatrix} (K^+, -\pi^+) \text{ for } Q=+1 \\ (\pi^-, K^-) \text{ for } Q=-1 \end{Bmatrix} \text{ for the } U\text{-spin doublets.}$$

the hypothesis (b) excludes the possibility that the $L_u^{(\pm)}$ belongs to 10 or $\overline{10}$.[7]

It will first be shown that the terms $L_1^{(-)}$ cannot cause the decay $K_1^0 \to \pi^0\pi^0$. Consider the 90° rotation about the (2) axis in the U-spin space. (This corresponds to the case $\theta=45°$ in the Cabibbo transformation.[1]) Under this transformation, $K_1^0=\frac{1}{2}\sqrt{2}(K^0-\bar{K}^0)$ is invariant and π^0 transforms into $\frac{1}{4}[\sqrt{3}(\sqrt{3}\pi^0+\eta)+\sqrt{2}(K^0+\bar{K}^0)]$, while $L_1^{(-)}$ transforms into $-\sqrt{2}L_1^{(0)}$ which is CP invariant and S and Q conserving. After this transformation, therefore, one has to consider two types of transition: $K^0 \to (\sqrt{3}\pi^0+\eta)K^0$ and $\bar{K}^0 \to (\sqrt{3}\pi^0+\eta)\bar{K}^0$. However, both of these transitions are forbidden since the CP-invariant interaction cannot connect the $K^0\bar{K}^0$ state and the π^0 or η state. Thus, one obtains

$$(L_u^{(-)}|K_1^0\pi^0\pi^0)=0 \quad \text{when} \quad u=1. \tag{2}$$

Here, $(L_u^{(-)}|K_1^0\pi^0\pi^0)$ represents the "amplitude" for the transition from the spurion corresponding to the decay Lagrangian $L_u^{(-)}$ to the $K_1^0\pi^0\pi^0$ system.

Next, let us consider the decay $K_1^0 \to \pi^0\pi^0$ induced by the term $L_1^{(+)}$. In this case, by the 90° rotation about the (1) axis in the U-spin space, the Lagrangian $L_1^{(+)} \propto \Psi(1,1)+\Psi(1,-1)$ is transformed into the Q- and S-conserving form $\sqrt{2}iL_1^{(0)}$. Under this transformation, $K_1^0 \to \frac{1}{2}i(\pi^0-\sqrt{3}\eta)$ and $\pi^0 \to \frac{1}{4}[\sqrt{3}(\sqrt{3}\pi^0+\eta)+i\sqrt{2}(K^0-\bar{K}^0)]$. Thus, instead of the original $K_1^0 \to \pi^0\pi^0$, one has to consider the transitions $(\pi^0-\sqrt{3}\eta) \to (\sqrt{3}\pi^0+\eta)\times(\sqrt{3}\pi^0+\eta)$ and $(\pi^0-\sqrt{3}\eta) \to (K^0\bar{K}^0+\bar{K}^0K^0)$, both of which are forbidden by the CP invariance of $L_1^{(0)}$. Hence,

$$(L_u^{(+)}|K_1^0\pi^0\pi^0)=0 \quad \text{when} \quad u=1. \tag{3}$$

Now, Eqs. (2) and (3) hold independently of what representation the Lagrangians (or spurions) $L_1^{(\pm)}$ belong to. On the other hand, in the decomposition of $8\otimes8\otimes8$, each of the totally symmetric representations 8, 27, and 64 appears only once, and therefore the transition amplitude $(L_u|K\pi\pi)$ depends only on the representation to which L_u belongs, but not on its u value. Moreover, the $K_1^0\pi^0\pi^0$ state actually contains the $u=1$, $u_3=\pm1$ components of all of the totally symmetric representations 8, 27, and 64. Hence, Eqs. (2) and (3) hold for all the spurions $L_u^{(\pm)}$ that belong to any representation and to any u value. This, in turn, leads to the following conclusion: *All the $K \to 2\pi$ decays are forbidden under the hypotheses* (*a*) *and* (*b*) described in the beginning (of course, in the limit of complete SU_3 symmetry of strong interactions).

Thus, as long as the usual current-current interaction or any other nonleptonic decay interactions that satisfy the hypothesis (b) are assumed, an attempt merely to step out of the octet hypothesis of nonleptonic decay Lagrangian cannot explain the $K \to 2\pi$ decays: They should be attributed to the symmetry-breaking[8] and/or electromagnetic interactions (and their large Q values).

Finally, let us suppose that the decay Lagrangian satisfies both the hypothesis (b) and the $\Delta I=\frac{1}{2}$ rule. The symmetry-breaking interaction ($\Delta I=0$) cannot affect the $\Delta I=\frac{1}{2}$ rule, and thus, even after inclusion of it, the $K^+ \to 2\pi$ decay is still forbidden. Next consider the effective Lagrangian L' which includes only the electromagnetic corrections but not the symmetry-breaking interactions. As for the terms of L' which belong to the representations 8, 27, and 64, the whole argument of the present note is valid and all the $K \to 2\pi$ decays are still forbidden. This is because the electromagnetic interaction has the character of a U-spin singlet and it is CP invariant. However, the L' also may contain the part $\bar{L}$ which belongs to a sum of 10 and $\overline{10}$ and whose $U_3=0$ partner $\bar{L}^{(0)}$ is CP invariant. As for this part, Eqs. (2) and (3) are still valid and $K_1^0 \to \pi^0\pi^0$ is forbidden, but this no longer implies the prohibition against the $K^\pm \to 2\pi$ and $K_1^0 \to \pi^+\pi^-$ decays. Therefore, if and only if such an $\bar{L}$ has sufficient magnitude, one may have a good chance to reconcile the $\Delta I=\frac{1}{2}$ rule with the observed ratio ($\sim$700) between the $K_1^0 \to 2\pi$ and the $K^+ \to 2\pi$ decay rates in such a way as has been pointed out by Cabibbo.[1] But again owing to the $\Delta U=0$ character of the electromagnetic corrections, the importance of this $\bar{L}$ (which has the $\Delta U=1$ character) means the importance of the $\Delta U=1$ part of the original $\Delta I=\frac{1}{2}$ Lagrangian. Also from this point, the octet hypothesis of nonleptonic decay Lagrangian seems very attractive.

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[7] This is because any *single* members of 10 or $\overline{10}$ cannot be an eigenstate of CP operation. Of course, an appropriate linear combination of the $Q=0$, $U_3=0$ members of 10 and $\overline{10}$ can be CP invariant. However, by the definition stated here, such cases are excluded from the present consideration. In this connection, see also the last part of the text, which discusses the electromagnetic corrections and mentions the importance of 10 and $\overline{10}$ in the $K^\pm \to 2\pi$ decay.

[8] Consider the interaction which is CP-invariant and behaves like $\Psi(1,0)$ under the rotation in the U-spin space. (The symmetry-breaking interaction would contain a part of this kind.) If this interaction is folded into any decay Lagrangian $L_u^{(\pm)}$ that belongs to a CP-invariant representation, then the resultant effective decay Lagrangian necessarily contains a part whose $U_3=0$ partner is not CP invariant (although the resultant Lagrangian itself is still CP invariant). Therefore, the inclusion of symmetry-breaking interaction, even in its lowest order, invalidates the whole argument of the present paper. In this connection, see also S. Okubo, Phys. Letters 8, 362 (1964). He has shown an example where a deviation from the complete SU_3 symmetry (precisely speaking, the introduction of derivative $K\pi\pi$ couplings and the $K-\pi$ mass difference) can give rise to the $K \to 2\pi$ decay.